CИM · METROPOLITAN BEOGRAD · UNIVERSITY GOCE DELČEV STIP · Universitatea Politehnica Timişoara

Co-funded by the
Erasmus+ Programme
of the European Union

| TOPIC PLAN | | |
|---|---|---|
| **Partner organization** | UNS | |
| **Topic** | Elementary functions with GeoGebra | |
| **Lesson title** | The properties of functions without derivative | |
| **Learning objectives** | Students will be able to analyze the properties of functions with parameters; <br><br> Students will acquire and deal with graphs of a function; <br><br> Students will be able to deal with different problems in everyday life, which require properties of given functions of a given function; <br><br> Students are encouraged to use technology and different software in their work, while considering problem-based situations. | **Strategies/Activities** <br> ☐Graphic Organizer <br> ☐ Think/Pair/Share <br> ☐Modeling <br> ☐Collaborative learning <br> ☐Discussion questions <br> ☐Project based learning <br> ☐Problem based learning <br><br> **Assessment for learning** <br> ☐Observations <br> ☐Conversations <br> ☐Work sample <br> ☐Conference <br> ☐Check list <br> ☐Diagnostics <br><br> **Assessment as learning** |
| **Aim of the lecture / Description of the practical problem** | The aim of the lecture is to make students able to analyze the properties of elementary functions to make base for further calculus tasks <br> The teacher gives the next tasks to the students: <br><br> 1. Use software, (GeoGebra) in order the examine the properties of linear, exponential and logarithmic functions <br> 2. Use software (GeoGebra) in order to examine the properties of power and trigonometric functions | |
| **Previous knowledge assumed:** | - functions <br> - algebraic equations | |

| Introduction / Theoretical basics | Introducrtion to GeoGebra<br>Definition of function.<br>The properties of function without derivatives,<br>Functions with parameters<br><br>a) $f(x) = kx + n, \ k, n \in R$<br>b) $f(x) = ax^2 + bx + c, \ a, b, c \in R$<br>c) $f(x) = a^{bx}, a > 0, b \in R$<br>d) $f(x) = \log_a bx, a > 0, b \in R$ | ☐Self-assessment<br>☐Peer-assessment<br>☐Presentation<br>☐Graphic Organizer<br>☐Homework |
|---|---|---|
| Action | For example, the analysis of given function in Geogebra depending on different parameters<br><br> | **Assessment of learning**<br>☐Test<br>☐Quiz<br>☐Presentation<br>☐Project<br>☐Published work |

Co-funded by the
Erasmus+ Programme
of the European Union

$$f(x) = a^{bx}, \ a > 0, \ b \geq 0$$

$$f(x) = a^{bx}, \ a > 0, \ b \leq 0$$
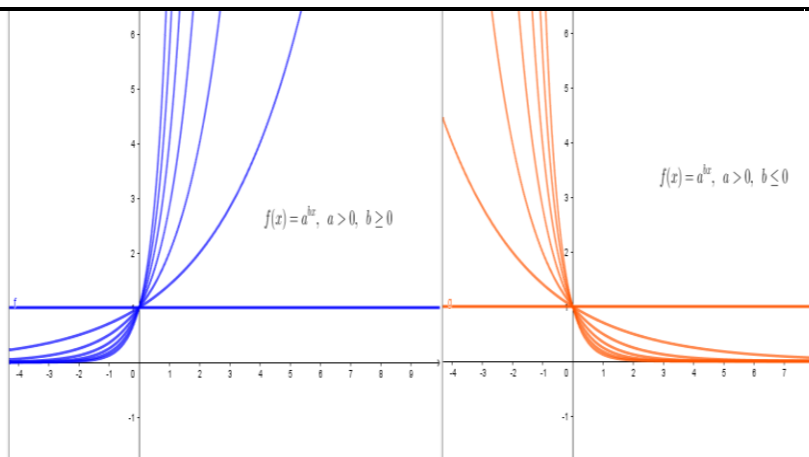
The following tasks are interesting too.
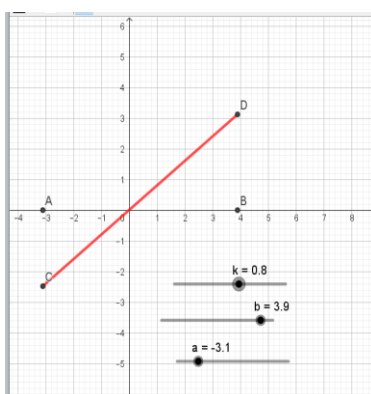
$f(x) = 0.8\,x, \quad x \in R, \quad Y = R, \quad$ Y je skup vrednosti funkcije
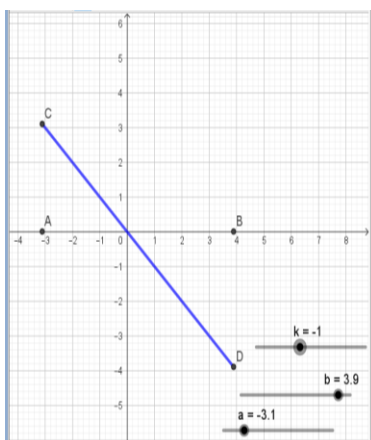
$f(x) = 0.8\,x, \quad [-3.1, 3.9]$

Na intervalu [-3.1,3.9], funcija f dostiže maksimum za x= 3.9

vrednost maksimuma f(3.9)=3.12

Na intervalu [ -3.1, 3.9], funcija f dostiže minimum za x= -3.1

vrednost minimuma je f(-3.1)=-2.48

Funkcija raste na intervalu [-3.1,3.9]

k = 0.8

b = 3.9

a = -3.1

$f(x) = -x, \quad x \in R, \quad Y = R, \quad$ Y je skup vrednosti funkcije

$f(x) = -x, \quad [-3.1, 3.9]$

Na intervalu [-3.1,3.9], funcija f dostiže maksimum za x= -3.1
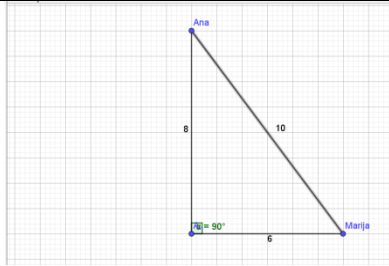
vrednost maksimuma f(-3.1)=3.1

Na intervalu [ -3.1, 3.9], funcija f dostiže minimum za x= 3.9

vrednost minimuma je f(3.9)= -3.9

Funkcija opada na intervalu [ -3.1,3.9]

k = -1

b = 3.9

a = -3.1

3. Let Marija and Ana start from the same place, and Marija is going, with a speed 6km per hour, to the north and Ana is going with a speed 8km per hour to the west. When the distance between them will be 2km.

| **Materials / equipment / digital tools / software** | *The materials for learning* are given as a part of references of the end from this topic plan;<br>*Equipment*: classroom, whiteboard, marker in different colours;<br>*Digital tools*: laptop, projector;<br>*Software*: Geogebra, | |
|---|---|---|
| **Consolidation** | With the given examples students can consider that the real functions and their graphs connecting with real different real life problems.<br>The students will be introduced to use GeoGebra package in solving mathematical problems connected with real life problems which will enable simultaneous multiple representation of mathematical concepts. This is important for mathematical thinking, in particular for advanced mathematical thinking in order to enable students' preparation for future work | |

**Reflections and next steps**

| **Activities that worked** | **Parts to be revisited** |
|---|---|
| Problem solving, collaboration, using technology<br><br><br>Your text here 2 | Depends on the students, in a conversation with students the teacher will realize the difficulties that students had and then revisit appropriate parts. |

**References**

**[1] Schmeelk, J., Takaci, D., takaci, A., Elementary analysis through examples and exercises, Springer 2010.**
[2] Stewart, J., Calculus, Thomson Learning, China, 2006.